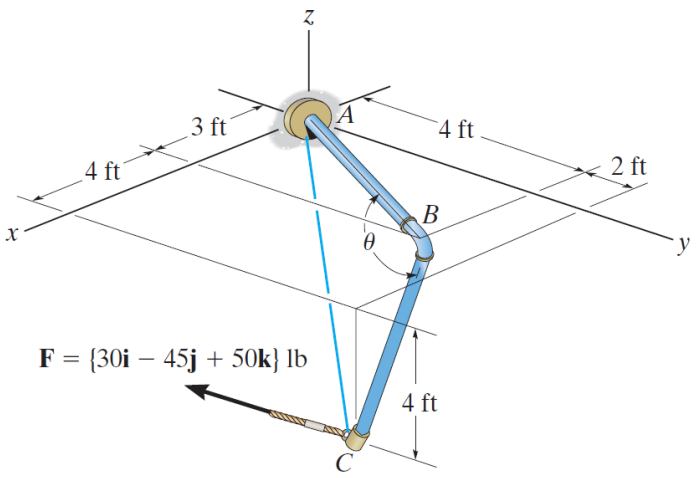
Problem 2-133

Determine the angle θ between the pipe segments BA and BC.



Probs. 2–131/132/133

Solution

Write the position vectors to the points A, B, and C.

$$\mathbf{r}_A = \langle 0, 0, 0 \rangle \text{ ft}$$

$$\mathbf{r}_B = \langle 3, 4, 0 \rangle \text{ ft}$$

$$\mathbf{r}_C = \langle 7, 6, -4 \rangle \text{ ft}$$

The unit vector going from B to A is

$$\hat{\mathbf{u}}_{BA} = \frac{\mathbf{r}_A - \mathbf{r}_B}{|\mathbf{r}_A - \mathbf{r}_B|} = \frac{\langle -3, -4, 0 \rangle}{\sqrt{(-3)^2 + (-4)^2 + (0)^2}},$$

and the unit vector going from B to C is

$$\hat{\mathbf{u}}_{BC} = \frac{\mathbf{r}_C - \mathbf{r}_B}{|\mathbf{r}_C - \mathbf{r}_B|} = \frac{\langle 4, 2, -4 \rangle}{\sqrt{(4)^2 + (2)^2 + (-4)^2}}.$$

Take the dot product of $\hat{\mathbf{u}}_{BA}$ and $\hat{\mathbf{u}}_{BC}$ to get the angle between them.

$$\cos\theta = \hat{\mathbf{u}}_{BA} \cdot \hat{\mathbf{u}}_{BC} = \frac{\langle -3, -4, 0 \rangle}{\sqrt{(-3)^2 + (-4)^2 + (0)^2}} \cdot \frac{\langle 4, 2, -4 \rangle}{\sqrt{(4)^2 + (2)^2 + (-4)^2}} = -\frac{2}{3}$$

Therefore, the angle between the pipe segments BA and BC is

$$\theta = \cos^{-1}\left(-\frac{2}{3}\right) \approx 132°.$$